An alternative method for calculating the energy of gravitational waves

Miroslav Súkeník and Jozef Sima

Slovak Technical University, Radlinského 9, SK-812 37 Bratislava, Slovakia

Abstract

In the expansive nondecelerative universe model, creation of matter occurs due to which the Vaidya metrics is applied. This fact allows for localizing gravitational energy and calculating the energy of gravitational waves using an approach alternative to the well established procedure based on quadrupole formula. Rationalization of the gradual increase in entropy of the Universe using relation describing the total curvature of space-time is given too.

As a source of gravitational waves, any physical system with time dependent mass distribution can be considered. The amount of such energy, emitted within a time unit is described by known quadrupole formula:

$$\frac{dE}{dt} = P_{qw} = -\frac{G}{45c^5}\dddot{K}_{\alpha\beta} \tag{1}$$

where $K_{\alpha\beta}$ is the tensor of quadrupole mass distribution in the source of emission. For $K_{\alpha\beta}$ it holds:

$$K_{\alpha\beta(t)} = \int \rho_{(t,x)}.(3X_\alpha X_\beta - \delta_{\alpha\beta}X_\kappa X^\kappa)\Delta V \tag{2}$$

In the expansive nondecelerative universe (ENU) model [1], the creation of matter and of gravitational energy simultaneously occur. The laws of energy conservation still hold since the energy of gravitational field is negative in ENU. The total energy of the Universe is thus exactly zero [2] and the Universe can continuously expand with the velocity of light c. This postulate is expressed in the ENU by equation:

$$a = c.t_c = \frac{2GM_u}{c^2} \tag{3}$$

where a is the gauge factor, t_c is the cosmological time, M_u is the mass of ENU.

In such a model, due to the matter creation the Vaidya metrics [3] must be used which enables to localise gravitational energy. Weak fields are in first approximation described by Tolman's relation [4]

$$\epsilon_g = -\frac{R.c^4}{8\pi.G} = -\frac{3m.c^2}{4\pi a.r^2} \tag{4}$$

in which ε_g is the density of the gravitational energy being emitted by a body with the mass m at the distance r, R denotes the scalar curvature (contrary to the more frequently used Schwarzschild metrics, in the Vaidya metrics, $R \neq 0$).

Using relations (3) and (4), equation expressing gravitational output P_g is obtained:

$$P_g = \frac{d}{dt}\int \epsilon_g dV = -\frac{m.c^3}{a} = -\frac{m.c^2}{t_c} \tag{5}$$

If the total mass of universe, M_u is substituted from (3) to (5), relation for the total output of the gravitational energy P_{tot} (its absolute value) by the Universe appears

$$P_{tot} = -\frac{c^5}{2G} \approx \left|2 \times 10^{52}\right| W \tag{6}$$

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This output is time independent and represents at the same time the upper limit of the gravitational energy output. No machine or physical phenomenon may create a higher output than that of P_{tot}. To illustrate the meaning of the above statement, radiant output of our galaxy is about 10^{37} W, number of galaxies in the perceptible part of the Universe is about 10^{11} which corresponds to the radiant output amounting 10^{48} W.

Sources of the gravitational waves can be both periodical and aperiodical. As an example of aperiodical sources, accelerated direct motion, burst of supernova or nonspheric gravitational breakdown can serve. Further we will focus our attention to periodical sources such as planets rotating around the Sun or a double star rotating around the common centre of inertia. The emitted gravitational energy cannot exceed P_{tot} value from (6). In any case, a condition (7) must be observed

$$P_{gw} \leq E_k.\omega P \leq \frac{c^5}{2G} \tag{7}$$

where P_{gw} is the energy of gravitational waves emitted within a time unit, E_k means the kinetic energy of a body moving on circular or elliptic orbit with the angular velocity ω. In these circumstances (the excentricity of an elliptic orbit is omitted in deriving the following relations), the ratio of the emitted gravitational energy P_{gw} and the kinetic energy E_k of such body will be comparable to that of E_k and P_{tot} (in the limiting case, both value are identical and equal to unity), i.e.

$$\frac{P_{gw}}{E_k.\omega} \approx \frac{E_k.\omega}{P_{tot}} \tag{8}$$

It follows from (8) that the output of emitted gravitational waves is

$$P_{gw} = -\frac{2G.E_k^2.\omega^2}{c^5} \tag{9}$$

The validity of equation (9) was tested on a system consisting of two bodies with the nearly identical masses

$$m_1 \doteq m_2 = m \tag{10}$$

These bodies rotate around the common centre of inertia on circular orbit with diameter r and angular velocity ω. The kinetic energy of either of the bodies is

$$E_k = \frac{1}{2}m.r^2.\omega^2 \tag{11}$$

Taking the identical masses of the bodies into account, both bodies must emit the same quantity of gravitational energy. Then it follows from (9), (10) and (11)

$$P_{gw} = -\frac{4G.E_k^2.\omega^2}{c^5} = -\frac{G.m^2.r^4.\omega^6}{c^5} \tag{12}$$

Quadrupole formula leads to equation

$$P_{gw} = -\frac{32G.r^4.\omega^6}{5c^5}\left(\frac{m_1.m_2}{m_1+m_2}\right)^2 \tag{13}$$

which can be, for cases where (10) and (13) hold, transformed into

$$P_{gw} = -\frac{8G.m^2.r^4.\omega^6}{5c^5} \tag{14}$$

Equation (14) differs from our relation (12) only in coefficient 8/5 which can be, bearing in mind the simplifications used in our derivation, assessed as an excellent agreement.

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Further domain of application of our approach lies in possibility to rationalize some questions which are still open, the entropy of the Universe in its begining being one of them. A problem arises [5] in interpretation of the individual elements in a simplified equation expressing the total curvature of space-time

$$R_{\bullet\bullet\bullet} = W_{\bullet\bullet\bullet} + R^o_{\bullet\bullet} g_{\bullet\bullet} \qquad (15)$$

where R is the Riemann tensor representing the total curvature of space-time, W is the Weyl's tensor describing deformation and slap forces, R^o is the Ricci's tensor, g is the metrics tensor.

Application of the Vaidya metrics manifests (4) that the scalar curvature decreases in time. Since this curvature is a reduction of the Ricci's tensor, the latter must decrease in time too. At identical conditions the Reimann's tensor is time independent which leads to conclusion stating that the Weyl's tensor must be gradually increasing in time and, in turn, the Universe had to start its expansion being in a highly ordered state, i.e. in the state with a minimal entropy [5].

Conclusions

The values of the energy of gravitational waves obtained using our alternative simple approach based on first approximation applied to the domain of weak gravitational fields are comparable to those derived by exact quadrupole relation. In addition to our previous results, the prospectiveness of the ENU model and Vaidya metrics is newly underlined.

References

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